Boundaries, Packing & Diversity – Spatial Scaling Laws in Squatter Settlements[1]

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1 INTRODUCTION

In this work it is shown that slums and squatter settlements are not merely disordered. Otherwise, they can be defined as complex structures, and such complexity can be quantified considering patterns of irregularity on the configuration. Particularly, it is made by analyzing the frequency of units (built and voids spaces) according to their sizes, i.e., by using scaling-laws.

In recent years a great deal of effort in pure and applied science has been devoted to the study of nontrivial spatial and temporal scaling laws which are robust, i.e. independent of the details of particular systems [1]. These studies at present involve a multitude of complex systems formed by a large number of small units communicating via short-range interactions and submitted to both deterministic rules and random influences. As examples of particular interest we can cite the study of the geometry of railway networks [2], as well as the study of the dynamics of traffic jams [3], and the modeling of urban growth patterns [4], among many others, all them exhibiting several types of scaling laws or power-law behavior. On the other hand, in the last few decades, the idea of disorder associated to the spatiotemporal configurations of cities has been replaced by the concept of complexity [5].

In the next section we report some results of this study on the geometry of squatter settlements. Our analysis focuses and quantifies in particular the fragmentation occurring in both the void and the built space of these urban structures. The data used in the first part of this present work came from an ensemble of nine squatter settlements or *"favelas"* distributed in different areas of the metropolitan region of Recife, on the northeastern coast of Brazil[3]. On the last section we present a brief comparison between these data and those from the configurational analysis on two squatter settlements situated along the Mathare Valley in Nairobi, central region of Kenya[4].

2 RESULTS AND DISCUSSION

In order to give a general view of the kind of urban structure we are dealing, we show in Figure 1 an image of the nine settlements examined in this first part of the present study. Each small cell of irregular shape in this figure represents an actual single habitation, and more precisely the space limited by roofs. As can be seen from these images, the settlements exhibit a seemingly disordered or spontaneous fragmented structure, characterized by the diversity of size of islands and the irregularity of their distribution and shape. These islands present a variable number s of habitations (s=1 means an isolated habitation, s=2 means a pair of contiguous houses, and so on). A careful examination of the Figure 1 reveals that s (size of islands) varies in the interval from 1 to 19. We can observe that there is a great number of small islands in each settlement, and as well a small number of big ones, a typical feature of complex systems. Another important characteristic of the settlements studied is that all them are embedded in urban networks and most of them submitted to very rigid boundary conditions. The development of these settlements occurs not as a spreading, but as a kind of packing process. Consequently, as the spatial limit of the settlement is previously defined, the diversity of size of its islands seems to be the response of the system to optimize the occupation.

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1 Work supported in part by CAPES, CNPq, FINEP, and PRONEX (Brazilian agencies).

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3 Source of data: URB–Recife (Recife Urbanization Enterprise).

4 Source of data: Housing Research and Development Unit - University of Nairobi – MATHARE VALLEY: A case study of Uncontrolled Settlement – 1971 – Nairobi.

Figure 1

The nine spontaneous settlements studied in the present work. See text, section 2 for details.

2.1 Geometry and Functions

To make clear to the general reader non-familiarized with the mathematical language, we in the following present a geometrical example for the statistical method suggested in this work. As example, it is shown a comparison between two hypothetical geometrical structures representing, respectively, the regular and irregular settlements (see figure 2).

Figure 2a

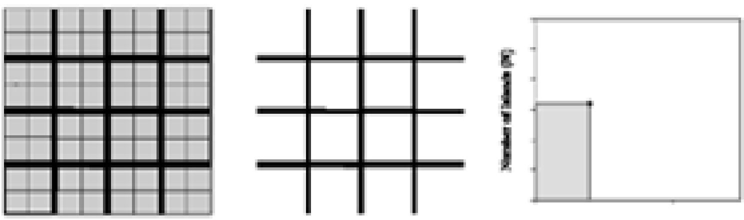
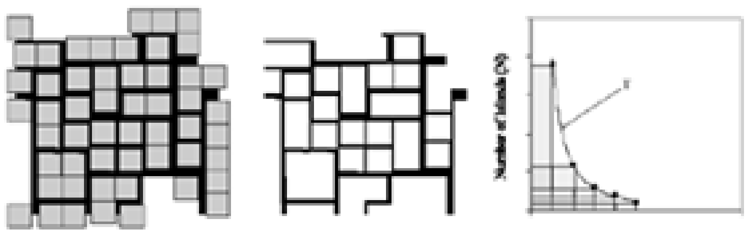


Figure 2b



Two hypothetical geometrical structures, representing a regular settlement (2a) and a irregular one (2b). Each one is represented by the map of islands, the grid of void spaces and the graph representing the number of islands (N) according their sizes (s). See text, section 2-A for details.

Figure 2a shows a regular grid, composed by a group of identical islands, distributed in a square area. It is really very simple to obtain global parameters from the local units of this regular structure. It is a simple procedure, as the linear function generates a regular shape. This kind of geometrical scheme, despite the great range of social constraints that it cause, has been very common through the history of urban planning not just due to the esthetical approach, but mainly because it can be easily reproduced, controlled, analyzed, measured and planned.

When analyzing figure 2b, which refers to the irregular settlement, one can note that the overall structure is not limited by a regular square and the local units (islands) are different in size to each other. To calculate the total built area (A) for the irregular settlement in figure 2b, we have to multiply the size (s) of each class of fragment (island) by the total number of fragments in that class, n(s), and by the average area (a_h) of an habitation and sum over all classes of size.

But, as we have argued, there is a difference between disordered systems and complex ones. When they are complex, they follow rules that can be measured by non-linear functions, and we suggest that such irregular structures (as the one presented on figure 2b, representing the squatter settlements of figure 1) have statistical properties of complex systems. So, to calculate the total number of habitations (hatched area on graph of figure 2b) of such squatter settlements we just need to know the curve of the function, i.e., the exponent T. And in this work we have found that all the settlement studied present a similar distribution of sizes, i.e., they follow non-linear functions with similar exponent T, whether to built or to void spaces.

2.2 Statistical analysis of the convex spaces

It is important to verify that the fragmentation observed on the maps of squatter settlements is not only characterized by the different sizes of islands, but also by the irregularity on shape of those islands and their informal distribution on space. Such irregularity is clearly observed when the void space is analyzed. The linear distribution of regular islands usually generate grids that can be easily modeled geometrically, while the irregular ones form a kind of deformed structure that seems to be quite disordered (see figure 2). But, as it will be shown in this first part of the analysis, such structure of void spaces can be described by non-linear geometrical patterns, directly related to the informality of the respective built structure.

So, to quantify the geometry of the settlements we investigated initially their void spaces, formed by alleys, courts and other interstitial regions surrounding the islands of habitations. To do this we performed a covering of this void space by a set of contiguous convex (polygonal) spaces as suggested by Hillier & Hanson [6]. According to these authors, a convex map for a structure defined on the plane is the least set of fattest spaces that covers the system. To adapt the convex space concept to the fragmentation approach applied here, we considered as void areas all the spaces without roofs. Thus, convex spaces, in our approach, are not only the common area, but also the areas behind fences (gardens, courts, etc). Figure 3 shows the convex maps of the nine settlement studied.

Figure 3

The covering of the void space of the nine settlements by convex (polygonal) spaces with a distribution of areas *f(a)*. See text, section 2-B for details.

Each covering in the present work is a family of fragments with a wide diversity of sizes, defined on this analysis by the variable area a_c. It is observed that such fragmentation, in a particular settlement, is characterized by a statistical distribution $f(a_c)$ which gives the frequency[5] of convex spaces within each interval[6] of area a_c. In spite of the nonuniqueness property of these coverings, we have observed that the distribution $f(a_c)$ is robust. Moreover, $f(a_c)$ has the scaling property and satisfies

$$f(a_c) \sim a_c^{-\beta}, \quad \beta = 1.6 \pm 0.2\,, \qquad (1)$$

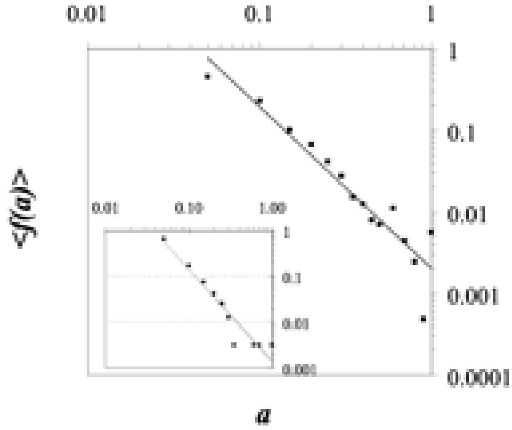
with the exponent beta independent of the settlement within the indicated statistical fluctuations. This scaling property suggest that the diversity of sizes of the resulting convex spaces follow a robust distribution of areas and indicates a great number of small convex spaces, and a small number of big ones.

Scaling or power-law behavior with nontrivial exponents is a characteristic of complex systems and means that the systems exhibiting such distributions have no characteristic scale or size [7]. The explanation of these scaling laws with nontrivial exponents is one of the great challenges in the study of complex systems. Figure 4 shows a log-log plot of the measured distribution $\langle f(a_c)\rangle$ of areas of convex spaces after dividing these spaces by classes of areas or bins and averaging over the entire ensemble of settlements given in Figure 3. In the horizontal axis of figure 4 the values a_c for areas were normalized with respect to the area of the largest convex space for each settlement. To help the reader in developing insight on the distribution of areas of convex spaces we exhibit in the inset of Figure 4 a nonaveraged $f(a_c)$ – in this case for the settlement 1 shown in Figure 3. As can be seen from these figures, both $\langle f(a_c)\rangle$ and $f(a_c)$ are essentially equal and have the same scaling exponent β, within the statistical fluctuations.

[5] Number of convex spaces divided by the total number of convex spaces to cover the entire void space.

[6] Since the variable a_c is in principle continuous, to arrive in equation 1 some sort of discretization is needed as usual.

Figure 4



Log-log plot of the measured, averaged, and normalised distribution of areas for convex spaces, $<f(a)>$, for the ensemble of nine settlements shown in Fig. 1. $<f(a)>\sim a^{-\beta}$, with $\beta = 1.6 \pm 0.2$ as represented by the continuous line in the main plot. The inset gives $f(a)$ for the settlement 2 shown in Fig.3. See text, section 2-B for details.

2.3. Statistical analysis for the islands of habitations

Inspired in statistical studies of fragmentation dynamics in physical systems and in order to obtain a more complete description of the geometrical properties of the urban structures studied here, we measured another distribution function, namely f(s), the frequency of islands with *s* contiguous habitations for each settlement. The discrete variable *s* gives a measure of the size or area of an island. This distribution is consequently defined for a region that is complementary to the void space defined in the previous paragraph.

We have found that the distribution f(s) also obeys a scaling relation; it is given by $f(s)\sim s^{-\tau}$, with $\tau = 1.6 \pm 0.2$ independently of the settlement. Thus, the value of the exponent τ is close to the value of β appearing in equation 1 within the statistical fluctuations. This means that, as the open structure, all the settlements analyzed present a similar distribution pattern for built elements, what suggests that the diversity of built spaces generate an equally diverse structure of void spaces. It is interesting to observe that such similarity of distributions between the convex spaces and the islands (defined by the similarity of exponents) suggests a strong relation between the built and void structures. Such correlation can also be supported by the similar degree of break-up verified on the settlements, that can be measured by the relation between the number of convex spaces and the number of buildings [6] in each settlement. We noted that practically all settlement studied present a high and similar level of breakup, i.e., the number of cells (houses) is close to the number of convex spaces. In a regular grid, the number of convex spaces is considerably reduced when compared to the number of cells of the system.

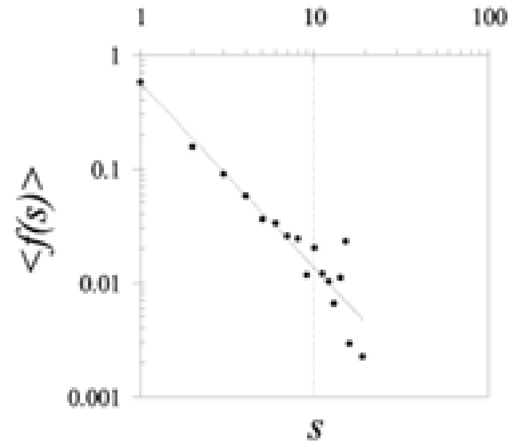
Scaling distributions of fragments with an exponent similar to this are commonly found in statistical models and in experiments of fragmentation dynamics of physical, chemical and ecological interest [8]. As an illustration we can cite that the distribution of size of fragments for different types of collapse of two-dimensional brittle solids as cement plates also presents a hyperbolic dependence with the size or area of the fragments with an exponent τ varying in the interval 1.5 to 1.8 [9]. The origin of this numerical value however is not yet clear. Another important illustration is the distribution of areas of the urban settlements around Berlin and London recently discussed by Makse et alli, when it is presented an exponent near to 1.9 [4]. However, reference 4 deals with a scaling distribution in a very large scale, when compared with the "microscopic" scale of lengths within the settlements examined in the present work, whose peculiarities are commented ahead.

The distribution of islands of habitations averaged on the ensemble of settlements shown in Figure 1 and normalized to the total number *N* of islands in each settlement, <f(s)>, is depicted in Figure 5. The continuous line in figure 5 represents the best fit

$$<f(s)> = <f(1)> . s^{-\tau} , <f(1)> = 0.49 \pm 0.08 , \tau = 1.6 \pm 0.2 \ ; 1<s<s_{max} \qquad (2)$$

where <f(1)> is the average number of islands with a single habitation, divided by the total number *N* of islands (<n(1)/N>). The largest size of islands (*smax)* present in the settlement studied *is 19.* The largest size, *smax*, can be a measure of the diversity of size of the islands, D (i.e. D can represent the number of different classes of size for the islands). In particular, *smax* converges to D when the level of complexity increases. From now on we will identify *smax* with D in all equations.

Figure 5



Log-log plot of the measured, averaged, and normalised distribution of sizes of islands with s habitations $<f(s)>$ for the ensemble of Fig.1. The continuous line represents the best adjust given by the power-law $<f(s)> = <f(1)>.s^{-\tau}$, with $<f(1)> = 0.49$, and $\tau = 1.6$.
See text, section 2-C for details.

The non-normalized distribution, $<n(s)>$, satisfies in general the constraint $<n(s=D)>=1$, that is, there is on the average a single island with the maximum size $s=D$ in the scaling. This result leads to $1=<n(1)>.D^{-\tau}$, or

$$<n(1)> = D^{\tau}, \qquad (3)$$

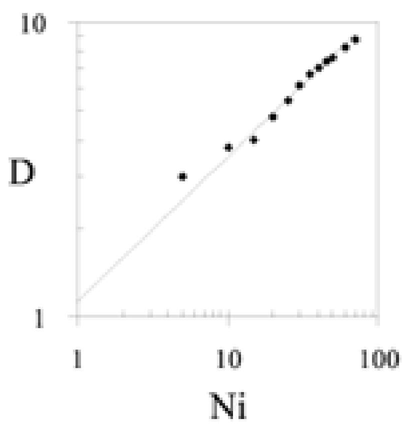
i.e., there is a simple relationship connecting the number of smallest islands $(s=1)$, $n(1)$, with the observed diversity of size of islands, D. When the first quantity increases (decreases), the second also increases (decreases), and the degree of coupling between $n(1)$ and D is controlled by the nontrivial exponent τ of the distribution. We can say, as a consequence, that $n(1)$ and D are two sides of a same coin. To increase the diversity of size of islands in a spontaneous settlement we need to increase the total number of islands, and this last quantity is mainly controlled by the number of isolated habitations n(1). The dependence of $n(1)$ with D given in (3) allows equation 2 to be written as

$$<n(s)> = <n(1)>\, s^{-\tau} = (s/D)^{-\tau}, \quad \tau = 1.6 \pm 0.2, \qquad (4)$$

what means that if one knows the diversity (generally equivalent to the size of its biggest island) of a squatter settlement, it is possible to estimate the number of islands with any size s, and vice versa.

The diversity of sizes as represented by the variable D here (see definition in the second paragraph of section 2-B) has been identified in studies of fragmentation dynamics with the overall complexity of the process or structure [7]. It is of interest to study how this diversity of size increases with the total number of islands, Ni. To examine this relationship we define for each settlement represented in Figure 1 a family of arbitrary balls with irregular contours in a such way that the family of balls contains successively $Ni=1, 5, 10, 15, 20, 25..., 100$ islands. The dependence of D with Ni is shown in Figure 6 in a log-log plot for all settlements; the diamond marks in this figure refer to the ensemble averages. The continuous line in Figure 6 represents the best fit $D = 1.1Ni^{0.49}$, for $1<Ni<80$. This relation says that the diversity of size of islands in a spontaneous settlement increase as the square root of the total number of islands. This last scaling law is in agreement with several recent experiments [9] and computer simulations [8] that have studied the evolution of the diversity in a fragmented system when its global size (e.g. the number of fragments) increases.

Figure 6



The diversity of sizes of islands of habitations, D, as a function of the number of islands, Ni. The continuous line represents the best fit $D = 1.1Ni^{0.49}$.
See text, section 4 for details.

3 OBTAINING PARAMETERS OF INTEREST

We have suggested that irregular settlements within the spatial constraints and statistical fluctuations described here, present the same pattern of fragmentation. These patterns occur on both the built and void spaces, and define their size distribution. We argue, as well, that it is possible to estimate global properties from general or local information of these settlements. Consequently, we can estimate global parameters as total area, number of houses and population. Obviously, the more developed the settlement (high level of diversity, great number of habitations, and consequently more data), the more precise will be the simulation.

The reader can observe that the distribution (4) is extremely economical: it depends only on a single parameter, namely D (the diversity of size of the settlement), With equation 4 valid on the average for all settlements studied, we can calculate several statistical parameters of interest after a simple integration, as we will show in the following.

The total area A occupied by the islands in a certain settlement is given by

$$A = \int_{s=1}^{s=D} \langle n(s) \rangle \cdot s \cdot a_h . ds = \int_{=1}^{s=D} (s/D)^{-\tau} \cdot s \cdot a_h \cdot ds = a_h \cdot \frac{[D^2 - D^\tau]}{2-\tau} \qquad (5)$$

where a_h is the average area per habitation. Since a_h can be easily guessed, equations 5 show that the total built area of a settlement is controlled by a single parameter, the diversity of size of island, D. Moreover, the total number of habitations in a settlement, $N_h=A/a_h$, takes the simple forms

$$N_h = \frac{[D^2 - D^\tau]}{2-\tau}, \text{ or} \qquad (6)$$

If the average population per habitation, p, is estimated by any mean, the total population, P, can be expressed in terms of the diversity D:

$$P = p.N_h = p.\frac{[D^2 - D^\tau]}{2-\tau}, \text{ or} \qquad (7)$$

Thus, the total population P is known if D is given and vice-versa. Another quantity of interest is the total number of islands in a given settlement, Ni, which is expressed as

$$N_i = \int_{s=1}^{s=D} n(s) \cdot ds = \frac{[D^\tau - D]}{\tau - 1}, \text{ or} \qquad (8)$$

In summary, if the distribution of islands or fragments for a particular fragmented system is known, all the statistical quantities of interest can be obtained. Hyperbolic distributions of the type given in (2) or (4) are specially robust and universal when they appear in a certain class of problems [1]. We conjecture that all spontaneous settlements that present the same kind of spatial constraints, are described by a single hyperbolic distribution of the type given in equation 4 with the scaling exponent τ assuming the robust value $\tau = 1.6 \pm 0.2$.

4 BOUNDARIES, PACKING & DIVERSITY

It is important to remember that the evolution process of the settlements analyzed in this work is not related to spreading, as the boundaries around the settlement limit the growth. Actually, it is a process of packing within a limited space for development. So, when the number of islands increase, the general size of the settlement remains the same and the density of the system raises, as each new building added to the system is submitted to the constraints of spatial availability.

If a settlement were composed only by islands of size 1 (isolated habitations), the resulting density would be considerably low, due to the large amount of void spaces, and we could conclude that in this case the occupation would be not optimized. On the other hand, if a structure were composed by a few amounts of really big islands, the rate of density would be really high. However, the resulting occupation would not be appropriate, due to some problems of access, privacy and salubrity caused by the lack of void spaces connecting houses. This would be, as well, a kind of non-optimized occupation. So, we can conclude that the best response to optimize the occupation of a decentralized system is through the diversity of size of its islands.

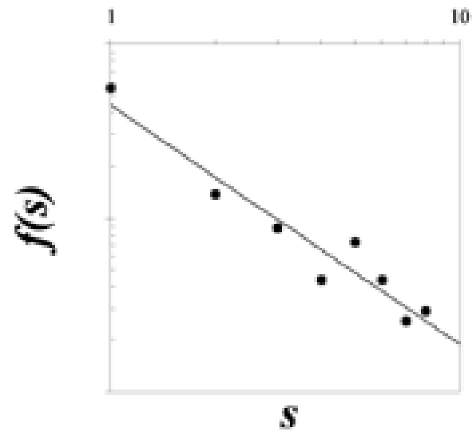
We can verify this relation about boundaries, packing and diversity by analyzing an actual example: the squatter settlements along the Mathare Valley in Nairobi, Kenia. In this case, we have compared the structure of two settlements (A and B), with the same age, but with distinct configurational patterns. In figure 7 it is shown a general view of the valley and the area around it. The settlements A and B are highlighted (the shaded and circled areas) in the general map and have their built structure detailed in specific plans, in which the shaded objects represent the islands of habitations. The settlement A is completely constrained by rigid boundaries: it is surrounded on the left and bellow by high-density urban structures; above by the Gitathuru river; bellow by the Juja road and on the right by property limits. On the other hand, the settlement B is completely free of boundaries, whether natural, urban or property related.

Figure 7

General view of the Mathare Valley (Nairobi – Kenya) and the area around it. The settlements A and B are highlighted (the shaded and circled areas) in the general map and have their built structure detailed in specific plans, in which the shaded objects represent the islands of habitations. See text, section 4 for details.

By analyzing the detailed maps of built structures, we clearly observe a great difference on the configurational structure of these settlements: the settlement B is practically composed by islands of isolated habitation [s=1], spread in a low-density structure; and the settlement A is highly packed (dense) and present islands with size from 1 to 8 (high level of diversity comparing to B). And what is more important: the frequency of islands [f(s)] according to the sizes follow a power law (see figure 8) defined by an exponent (1.4) similar to the exponent found for the settlements in Brazil. These results seem to indicate a deterministic order, where boundary is the cause, packing is the effect and diversity is the route.

Figure 8



Log-log plot of the measured, and normalised distribution of sizes of islands with *s* habitations $\langle f(s) \rangle$ for the settlement A ensemble of Fig.7. The continuous line represents the best adjust given by the power-law $\langle f(s) \rangle = \langle f(1) \rangle . s^{-\tau}$, with $\langle f(1) \rangle = 0.44$, and $\tau = 1.4$.
See text, section 4 for details.

5 CONCLUSIONS

We have shown that there are robust (possibly universal) distribution functions associated to the fragmented structures of these spontaneous settlements. Using these distribution functions, many statistical quantities of interest can be obtained. In particular, we have stressed the importance between the variable diversity, packing and boundaries. We conjecture that the hyperbolic distribution functions given in equations 1 and 4 are robust and universal, that is, they control the statistical aspects discussed here in all squatter settlements irrespective cultural particularities. An important characteristic of the settlement studied is that all them are embedded in urban networks and most of them submitted to very rigid boundary conditions. The development of these settlements occurs not as a spreading, but as a kind of packing process. Consequently, as the spatial limit of the settlement is previously defined, the diversity of size is the response of the system to optimize the occupation. If there is not rigid boundaries, the response is trivial: houses distributed by a disordered way, isolated in islands of size 1. Anyway, a detailed analysis of other spontaneous settlements in different regions is needed to test the robustness of the present conjecture.

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